2.1, 2.2 Self-Assess

<u>Context</u> of Problem

3pt problem, write $\frac{2}{3}$ OR $\frac{3}{3}$ OR $\frac{2.5}{3}$ next to problem.

Add up all the points.

$\frac{25}{10}$ at the top.

2.4 Properties of Quadratic Functions

These materials are brush-up for the "Are You Prepared?" questions:

- Intercepts (Foundations, Section F.2, pp. 10–11)
- Graphing Techniques: Transformations (Section 1.5, pp. 85–94)
- Completing the Square (Section A.4, pp. A35–A36)

$x=2$ is axis of symmetry

OBJECTIVES

1 Graph a Quadratic Function Using Transformations
2 Identify the Vertex and Axis of Symmetry of a Quadratic Function
3 Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts
4 Find the Maximum or Minimum Value of a Quadratic Function

'Are You Prepared?'

1. List the intercepts of the equation $y = x^2 - 9$. (pp. 10–11)

2. List the intercepts of the equation $y = 2x^2 + 7x - 4$. (pp. 10–11)

$= 2x^2 + 8x - 1x - 4$

$= 2x(x+4) - 1(x+4) = (x+4)(2x-1) = 0$

$\Longrightarrow x \in \{-4, \frac{1}{2}\}$

$(-4,0), (\frac{1}{2},0)$

$(0,-4)$

3. To complete the square of $x^2 - 5x$, you add the number _____. (pp. A35–A36)

4. To graph $y = (x-4)^2$, you shift the graph of $y = x^2$ to the RIGHT a distance of 4 units. (pp. 86–88)

Concepts and Vocabulary

5. The graph of a quadratic function is called a(n) _____.
6. The vertical line passing through the vertex of a parabola is called the _____.
7. The x-coordinate of the vertex of $f(x) = ax^2 + bx + c$, $a \neq 0$, is _____.
8. *True or False*: The graph of $f(x) = 2x^2 + 3x - 4$ opens up.
9. *True or False*: The y-coordinate of the vertex of $f(x) = -x^2 + 4x + 5$ is $f(2)$.
10. *True or False*: If the discriminant $b^2 - 4ac = 0$, the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$, will touch the x-axis at its vertex.

Graph a Quadratic Function Using Transformations

Let $f(x) = x^2$ be our basic function.

I had you do this on a quadratic function $g(x) = a(x - h)^2 + k$, which is of the form

$f(x)=x^2$

$$a\,f(x-h)+k$$

Vertex! $(-2, 11)$

opens down

$$g(x) = -(x+2)^2 + 11$$

In terms of $f(x) = x^2$ and the formulation above, this is

$$-1f(x-(-2)) + 11.$$

I will walk you through this one from the test. Example 1, is another good walk-through on how to graph $a(x - h)^2 + k$.

It's the *algebra* in Example 1 that leads from $ax^2 + bx + c$ to $a(x - h)^2 + k$ that students find "tricky."

* $(3x-2)(2x+5) = 6x^2 + 11x - 10$

$$= 6\left(x^2 + \frac{11}{6}x + \left(\frac{11}{12}\right)^2\right) - 10 - 6\left(\frac{11}{12}\right)^2$$

$$\frac{\frac{11}{6}}{\frac{2}{1}} = \frac{11}{6}\cdot\frac{1}{2}$$

$$\frac{11}{12} \to \left(\frac{11}{12}\right)^2$$

scratch

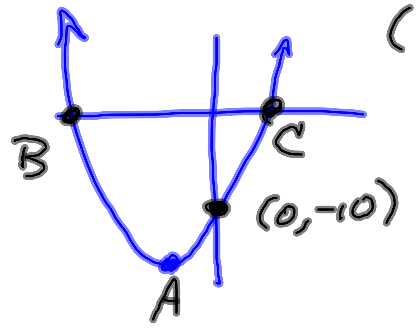
$$-10 - 6\left(\frac{121}{144}\right)$$

$$= -10 - \frac{121}{24} = \frac{-240-121}{24} = -\frac{361}{24}$$

$$= 6\left(x + \frac{11}{12}\right)^2 - \frac{361}{24}$$

Parabola with vertex

$$(h,k) = \left(-\frac{11}{12}, -\frac{361}{24}\right) = A$$



x-intercepts:
From Factorization, we have $(3x-2)(2x+5) \overset{SET}{=} 0$

$\Rightarrow x = 2/3$ or $x = -5/2$

$B = (-5/2, 0)$, $C = (2/3, 0)$.

Another way:

$$f(x) = 6\left(x + \frac{11}{12}\right)^2 - \frac{361}{24} \overset{SET}{=} 0$$

$$6\left(x + \frac{11}{12}\right)^2 = \frac{361}{24}$$

$$\left(x + \frac{11}{12}\right)^2 = \frac{361}{144}$$

$$\sqrt{\left(x+\frac{11}{12}\right)^2} = \sqrt{\frac{361}{144}} = \frac{\sqrt{361}}{\sqrt{144}} = \frac{19}{12}$$

$$\left|x+\frac{11}{12}\right| = \frac{19}{12}$$

$$x+\frac{11}{12} = \pm\frac{19}{12}$$

$$x = -\frac{11}{12} \pm \frac{19}{12}$$

$$\frac{-11+19}{12} = \frac{8}{12} = \frac{2}{3}$$

$$\frac{-11-19}{12} = \frac{-30}{12} = \frac{-5}{2}$$

This recipe is given in your textbook. It's pretty good.

Steps for Graphing a Quadratic Function $f(x) = ax^2 + bx + c$, $a \neq$ (

Option 1

Step 1: Complete the square in x to write the quadratic function in the form $f(x) = a(x - h)^2 +$

Step 2: Graph the function in stages using transformations.

Option 2

Step 1: Determine the vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Step 2: Determine the axis of symmetry, $x = -\frac{b}{2a}$.

Step 3: Determine the y-intercept, $f(0)$.

(a) If $b^2 - 4ac > 0$, the graph of the quadratic function has two x-intercepts, which are found by solving the equation $ax^2 + bx + c = 0$.

(b) If $b^2 - 4ac = 0$, the vertex is the x-intercept.

(c) If $b^2 - 4ac < 0$, there are no x-intercepts.

Step 5: Determine an additional point by using the y-intercept and the axis of symmetry

Step 6: Plot the points and draw the graph.

If you complete the square, you get the *-b*/2*a* and *f*(*-b*/2*a*) for *free*! So don't waste time re-calculating them!!!

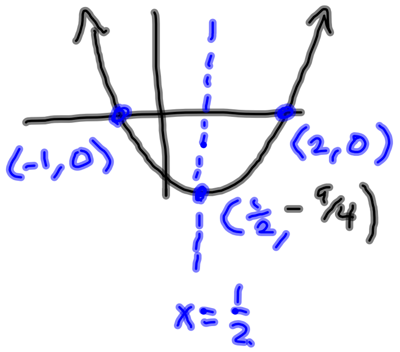
I *rarely*, if *ever*, use $-\dfrac{b}{2a}$ to find h. I just let h fall in my lap for free, after I've completed the square. Nice thing about that is that I don't mess with evaluating

$$k = f\left(-\frac{b}{2a}\right)$$

Instead, I just read k from the completed-square version of the expression. In most college algebra circumstances, completing the square is quicker and cleaner than evaluating...

$$\begin{aligned} f(x) &= a(x-h)^2 + k \\ &= a\left(x + \frac{b}{2a}\right)^2 + f\left(\frac{b}{2a}\right) \end{aligned}$$

$(x-2)(x+1) = f(x)$ $= x^2 - x - 2 = f(x)$



$a = 1,\ b = -1,\ c = -2$

$-\frac{b}{2a} = -\frac{-1}{2} = \frac{1}{2} =$

$f(-\frac{b}{2a}) = f(\frac{1}{2}) = (\frac{1}{2})^2 - \frac{1}{2} - 2$

$= \frac{1}{4} - \frac{1}{2} - 2$

$= -\frac{1}{4} - \frac{8}{4} = -\frac{9}{4}$

So, $f(x) = x^2 - x - 2$

$= (x - \frac{1}{2})^2 - \frac{9}{4}$

$x^2 - x - 2 = x^2 - x + (\frac{1}{2})^2 - 2 - \frac{1}{4} = \boxed{(x - \frac{1}{2})^2 - \frac{9}{4}}$

$1 \to \frac{1}{2} \to (\frac{1}{2})^2$

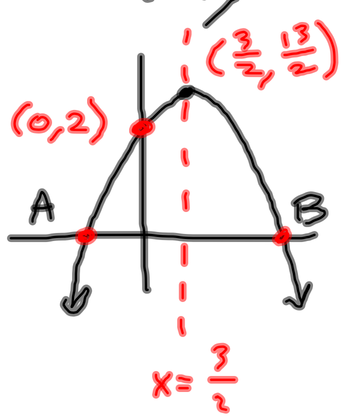
32. $f(x) = -2x^2 + 6x + 2$

$= -2\left(x^2 - 3x + \left(\frac{3}{2}\right)^2\right) + 2 + 2\left(\frac{9}{4}\right)$

$= -2\left(x - \frac{3}{2}\right)^2 + \frac{4}{2} + \frac{9}{2}$

$= -2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$

$(h, k) = \left(\frac{3}{2}, \frac{13}{2}\right)$



$= -2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$ SET $= 0$

$-2\left(x - \frac{3}{2}\right)^2 = -\frac{13}{2}$

$\left(x - \frac{3}{2}\right)^2 = \frac{13}{4}$

$x - \frac{3}{2} = \pm\sqrt{\frac{13}{4}} = \pm\frac{\sqrt{13}}{2}$

$x = \frac{3}{2} \pm \frac{\sqrt{13}}{2}$

$\boxed{A = \left(\frac{3 - \sqrt{13}}{2}, 0\right),\ B = \left(\frac{3 + \sqrt{13}}{2}, 0\right)}$

The zeros of $f(x)$ are $\frac{3 \pm \sqrt{13}}{2}$, so

$f(x) = -2\left(x - \frac{3 + \sqrt{13}}{2}\right)\left(x - \frac{3 - \sqrt{13}}{2}\right)$

is its factored form.

$-\frac{b}{2a}$ way

$-2x^2 + 6x + 2$

$a = -2,\ b = 6,\ c = 2$

$-\frac{b}{2a} = \frac{-6}{2(-2)} = \frac{-6}{-4} = \frac{3}{2}$

$f\left(-\frac{b}{2a}\right) = -2\left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right) + 2$

$= -2\left(\frac{9}{4}\right) + 9 + 2$

$= -\frac{9}{2} + 11 = -\frac{9}{2} + \frac{22}{2} = \frac{13}{2}$

$(h, k) = \left(\frac{3}{2}, \frac{13}{2}\right)$

$b^2 - 4ac = 36 - 4(-2)(2)$

$= 36 + 16 = 52$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-6 \pm \sqrt{52}}{2(-2)} = \frac{-6 \pm \sqrt{52}}{-4}$

$= \frac{-6 \pm 2\sqrt{13}}{-4} = \frac{2(-3 \pm \sqrt{13})}{-4}$

$= \frac{-3 \pm \sqrt{13}}{-2}$

whole class burned me.

$2 \overline{)52}$
$2 \overline{)26}$
13

$52 = 2^2 \cdot 13$

$\sqrt{52} = \sqrt{2^2 \cdot 13}$

$= \sqrt{2^2}\sqrt{13}$

$= 2\sqrt{13}$

In Problems 35–52, (a) graph each quadratic function by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any. (b) Determine the domain and the range of the function. (c) Determine where the function is increasing and where it is decreasing.

#s 35 - 52: (a) Graph each quadratic function by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, *y*-intercept, and *x*-intercepts (if any).

(b) Determine the domain and range of the function.

(c) Determine where the function is increasing and where it is decreasing.

38. $f(x) = -x^2 + 4x$

49. $f(x) = 3x^2 + 6x + 2$

In Problems 53–58, determine the quadratic function whose graph is given.

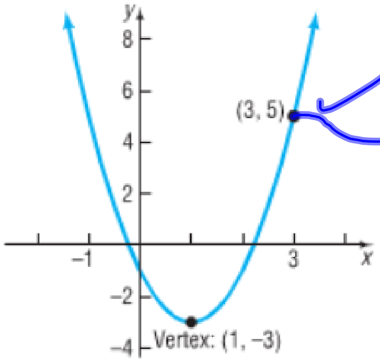
Vertex is (h, k)

Cool puzzles!

$f(x) = a(x-h)^2 + k$

$= a(x-1)^2 - 3$

57.



Says $f(3) = 5$

$f(3) = a(3-1)^2 - 3 = 5$

$a(2)^2 - 3 = 5$

$4a - 3 = 5$

$4a = 8$

$a = 2$

So, $f(x) = 2(x-1)^2 - 3$

#s 75 - 79 are a preview of the

Factor Theorem: If you know the factors of a polynomial, you know its zeros, and conversely.

That's the punchline of these 5 exercises.

You tell me the zeros, and I can *build* you a polynomial that has those zeros!

Read the instructions to #s 75 and 76 carefully, and see how I use them, here.

#75: Find a quadratic function $f(x) = ax^2 + bx + c$ whose x–intercepts are (2, 0) and (–5, 0). Build one where $a = 1$; howzabout building one with $a = 3$?

$f(x) = 1(x-2)(x+5)$

$= x^2+3x-10$ does the trick.

Need a leading coefficient of 3?

No prob: $3(x-2)(x+5)$

$= 3(x^2+3x-10)$

$= 3x^2+9x-30$

Recall from previous work this a.m.:

32. $f(x) = -2x^2 + 6x + 2$

The zeros of $f(x)$ are

$\frac{3 \pm \sqrt{13}}{2}$, so

$f(x) = -2\left(x - \frac{3+\sqrt{13}}{2}\right)\left(x - \frac{3-\sqrt{13}}{2}\right)$

is its factored form.